# Investigation: Product of Polynomials – Possible Responses

**Component 1**

In this component you will consider graphs of cubic polynomials.

1. Consider the family of curves of the form where are real numbers .
2. i. By selecting your own values for where , sketch 3 cubic graphs of the above form. Label your axial intercepts with coordinates. Also write the equation for each corresponding graph.

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1. Comment on any similarities/differences between your graphs.

Similarities:

* They all have 3 -intercepts
* They all have 2 turning points
* They all have the same shape

Differences:

* Some have positive -intercepts while others have negative -intercepts
* Some have all positive -intercepts, some have all negative -intercepts, and some have both positive and negative -intercepts

1. Discuss how affect the key features of the graph.

The values pf correspond to the values of the -intercepts

1. What happens to the shape and key features of the cubic graph if ? Investigate. Provide 3 examples to support your ideas/conjectures. Label your axial intercepts with coordinates. Also write the equation for each corresponding graph.  
   Comment on any similarities/differences. Try to generalise your observations.

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Similarities:

* All have the same shape
* All have just one -intercept
* All have a stationary point of inflection at the -intercept

Differences:

* Some have positive -intercepts while others have negative -intercepts
* Some have positive -intercepts, some have negative -intercepts

Generalisations:

* All graphs of the form will have a stationary point of inflection at the -intercept
* The graph/stationary point of inflection/-intercept will move to the right if and will move to the left if
* The -interept of will be positive if and negative if

1. What happens to the shape and key features of the cubic graph if ? Investigate. Provide 3 examples to support your ideas/conjectures. Label your axial intercepts with coordinates. Also write the equation for each corresponding graph.  
   Comment on any similarities/differences. Try to generalise your observations.

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Similarities:

* All have the same shape
* All have just two -intercept
* Some students may use CAS technology to identify that all of these graphs will have a (non-stationary) point of inflection

Differences:

* Some have positive -intercepts while others have negative -intercepts
* Some have positive -intercepts, some have negative -intercepts

Generalisations:

* All graphs of the form will have a turning point at (or at the point ), and the graph will cut through the -axis at (or the point ).
* The graph will have a positive -intercept if is negative, and a negative -intercept if is positive

1. Consider the family of curves of the form ,   
   where are non-zero real numbers.  
   Investigate how affect the shape and key features of the graph.  
   Provide examples to support your ideas/conjectures. Summarise/describe your observations, and try to generalise your observations.

* corresponds to the -intercepts
* All graphs will pass through the -intercept
* will determine the number of intercepts of the graph
  + will give one -intercept at
  + will give two -intercepts
  + will give three -intercepts

**Component 2**

In this component you will consider graphs of the form , where is non-zero real number and

1. Consider the case where .
2. Sketch the following graphs, labelling all key features.

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1. Comment on any similarities/differences between your graphs.

Similarities:

* All graphs (except for the case of ) pass through the points and /have two -intercepts
* The graphs of and & (i.e. ) have a cubic shape.
* The graphs of and & have a point of inflection at one of the -intercepts (at the intercept whose factor has degree 3).
* There is a turning point when at least one of the powers is greater than or equal to 2 (2 or more).
* There is a stationary point of inflection when one of the powers is 3

Differences:

* ~~Some have positive -intercepts while others have negative -intercepts~~
* ~~Some have positive -intercepts, some have negative -intercepts~~

1. Discuss how and affect:

* The behaviour/shape of the graph. Provide examples to support your ideas/conjectures (select a different value of ).
* The number and nature of any turning points/point of inflection. Provide examples to support your ideas/conjectures (select a different value of ).
* affect the behaviour of the graph at the -intercepts
* When or is odd, the graph cuts through the -intercept
  + If or is 3, there is also a stationary point of inflection at the intercept that has the power of 3
* When or is even, the graph will have a turning point at the -interept that corresponds to that power (factor).

1. What would happen to the graph and key features if was negative? Investigate.  
   Students could try a particular value of (or values of ), compare with their earlier results and then notice any patterns emerging to generalise their results/observations.

Consider

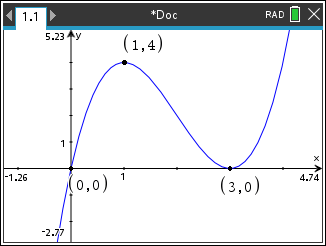
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* For , the -intercepts are now located at and
* The -intercepts are located at and
* Graphs where is even appear to be reflected in the -axis, however this is not the case for graphs where is odd (Note: this could be extended into exploring odd and even functions)
* The number of turning points and points of inflection are the same as when is positive, however their locations are different

1. Create an equation of a graph of the form that satisfies the following conditions:

Graph 1: Has an -intercept and turning point at

One possible answer:



Anything of the form where and will work

Graph 2:

* Has at least 2 turning points
* Has a negative -intercept

Possible answers could include:

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Graph 3:

* Has a stationary point of inflection
* Has a negative -intercept

Possible answers could include:

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**Component 3**

In this component you will consider graphs of the form ,   
where are non-zero real numbers and .

1. Consider the family of curves of the form where .  
   Investigate how and affect:

* The location and number of axial intercepts
* The behaviour of the graph
* (The number of turning points/stationary points of inflection)

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**The location and number of axial intercepts**

* In all cases, there is an -intercept at (at the point )
* In all cases, there is a -intercept at (at the point )
* In all cases, there two positive -intercepts
* When or (or ) there are only 2 -intercepts, located at and or
* When ( there are three -intercepts, since the factors are one that is linear and one that is a difference of perfect squares
* When , the -intercepts are at and
* When , the -intercepts are at and

**The behaviour of the graph**

* When or , the graph has a quadratic or quartic shape ( shape)
* When , the graph has a cubic shape

**The number of turning points/stationary points of inflection**

* When , there is one turning point
* When , there are two turning points (and one non-stationary point of inflection)
* When , there is one turning point and one point of inflection

1. Consider the family of curves of the form where and is a non-negative real number.  
   Investigate how and affect:

* The location and number of axial intercepts
* The behaviour of the graph
* (The number of turning points/stationary points of inflection)

**End of Investigation**